

XI Crisis of thought of the theorists of the foundations of mathematics¹

April 2022. A supposedly consistent theory of the transfinite is postulated as part of the foundations of mathematics, despite being confirmed already a century ago to be inconsistent with reality. This thinking of the theorists of foundations of mathematics reveals disregard for the criteria of science. A theory that denies realness is necessarily and demonstrably wrong.

The empty set, that is postulated to justify the transfinite, fails in reality. The insistence on the infinite subdivision in contradiction to Planck, the fiction of insignificance of the syntax of mathematical logic, the failure to resolve the antinomies and the refusal to perceive the disproof of Gödel's incompleteness theorems demonstrate a disruption of thinking. The foundational crisis of mathematics, that evolved before the 20th century for decades, was not overcome by the measures taken to overcome it, but exacerbated.

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1. Introduction

1.1. The exclusion of inconsistencies between theory and reality is an unavoidable demand of science. Theories may reach beyond reality as a prerequisite for further development, but once they are shown to contradict reality, they are disproved. In the following it will be discussed whether the foundations of mathematics meet this need.

1.2. Contradictions do not exist in nature. The inconsistency of antinomies is the result of pseudo-logical misconstructions of the human mind that can be resolved. Mathematicians and logicians classified the antinomies of set theory as unsolvable, the fatal consequences for the foundations of mathematics are argued.

2. The transfinite

Set theory of Georg Cantor belongs to the foundations of mathematics. His perception of the infinite, the transfinite, is distinguished by limits and levels. At first he claimed conformity with nature, but then retreated to the position of "conceivable and not objectionable".

Line segments can be mapped to numbers. Article II demonstrates that transfinite segments and numbers are in contradiction to reality. David Hilbert has been very clear on this in 1926 in *Über das Unendliche*. **"The infinite is nowhere realized; it does not exist in nature, nor is it permissible as a basis for our intellectual thinking.** However, the transfinite is **an "idea secured independently of all logic"**. Hilbert confirms a "successful, complete calculus" for Cantor's theory, free of contradictions. He also approved the axiomatic system ZFC, that founded Cantor's considerations axiomatically.

Hilbert thus notes a contradiction to reality, but insists that the mathematical theory is consistent. Even today, no one will claim that limits and levels in infinity can be brought into agreement with physics and astronomy without contradiction, but the mathematical theory is kept hold of.

In fact, it can be proven that Cantor's theory contains serious errors and that ZFC is also contradictory (Articles III, IV). Moreover Cantor's justification of the transfinite is disproved by

¹ The topic including the bibliographical references are part of the book by Gert Treiber, *"Nichts", Krise und reEvolution der Grundlagen der Mathematik*, Cuvillier Verlag, 2020.
The articles of this outline the main points of the topic .

ZFC, introducing the empty set as a necessary prerequisite for the transfinite set. The empty set does not exist in Cantor's theory.

Hilbert moreover tries to legitimate his conviction philosophically. He claims the validity of an "idea that surpasses all experience", demanding agreement with Immanuel Kant. The latter actually called for synthetic a priori judgement of the mind, independent of empirical experience, and classified into categories. Kant, however, stated: "The category has no other use for the knowledge of things than its application to objects of experience". Hilbert did not understand Kant.

A supposedly consistent theory of the transfinite is postulated as part of the foundations of mathematics, although its contradiction with reality has been accepted a century ago.

3. The 0 and the empty set

3.1. Today, 0 is accepted as a number without doubt . In fact, it cannot be a number, but represents "nothing" of numbers, i.e. "no digit" and "no number" (art. I). Applied mathematics of "number 0" is, however, consistent, but "number 0" in the foundational theory is viewed critically. A contradiction between set theory and arithmetic is accepted without reflection. **"Number 0" as an element increases a set, but the summand "number 0" leaves the sum unchanged.** Revised set theory resolves this contradiction, **"no number", 0, can neither increase a sum nor a set** (art. II).

3.2. Fraenkel demanded validity of the axiomatic system of set theory not only for abstract objects, but also for "apples, pears and apricots". **Current theory postulates that the empty set $\emptyset$ as an element increases any set. But the empty set without apples cannot increase a set of apples** (article IV). The empty was not reviewed in reality, it was not perceived that it cannot exist. The sign $\emptyset$ indeed represents "no element" and "no set".

4. The infinite subdivision

Since Euclid, the infinite subdivision of line segments has been premised, with the consequence of mapping to real numbers with an infinite number of digits. In 1903, Hilbert also adopted this assumption in his *Grundlagen der Geometrie*. In 1899, however, **Max Planck detected units such as the Planck length, which cannot be undercut**. Hilbert admitted that infinite subdivision contradicts reality, but drew no conclusions for geometry and real numbers from it. Mathematics is in contradiction to physics. **Potentially infinite converging sequences and confining limits can no longer be justified. They are replaced by converging finite sequences with limitation** (Article VI). Irrational numbers can no longer be justified, this also applies to the limit of the differential and integral calculus.

5. Fiction of the insignificance of the syntax of mathematical logic

To re-found mathematics, Hilbert developed the formalism of mathematical logic in order to overcome the antinomies. "There is a perfectly satisfying way of escaping the paradoxes. **The sign has no meaning**". **The formulas** of the syntax of mathematical logic **also have no meaning** and follow one another according to purely formal rules. It is only through semantics that "with regard to content considerations a higher level is achieved". The intrinsic mathematics is established "purely syntactic, free from semantic interpretation".

Critics rejected the "completely meaningless intrinsic mathematics. Don't let yourself be deceived about the lack of content in this Hilbertian mathematics. Signs that mean nothing are signs divested of their pointing function, i.e. no signs at all."

That precisely defined but meaningless syntactic formulas and rules can be formed from meaningless signs is an unreal fiction. Nevertheless, this interpretation of the syntax belongs to the authoritative theory of mathematical logic. In fact, its importance is justified by satisfying every semantic interpretation in the highest abstraction (Article VIII).

6. The antinomies

The antinomies of set theory, especially Russells antinomy, provoked a foundational crisis of mathematics at the turn of the 19th to the 20th century. **The failure to resolve the antinomies also proves a grievous crisis of thinking:**

- a. **Russells antinomy** ascribes contradictory properties to the "set of all sets that do not contain themselves". The book "Nichts" demonstrates that the contradiction can be resolved, the foundational crisis could have been avoided.
- b. Russell and Whitehead's program of founding mathematics on logic through the **Principia Mathematica** mainly failed because of the antinomy.
- c. Gottlob Frege had to give up his definition of the set because of Russells antinomy. To date there is **no definition of the set**, it is only required that it should satisfy the axioms.
- e. Cantor assumed that exclusively sets are existent. The antinomy of Burali-Forti revealed that the infinite sequence of ordinal numbers does not allow for the union to a set. Even Cantor conceded this fact in a letter. In the axiomatic system ZFC, on the other hand, it is disputed. **"Objects other than sets do not exist for us at all.** Primarily, the formation of paradoxical sets (antinomies by Burali-Forti, Russell, etc.) is excluded by our axioms", Fraenkel records. In the same article, however, he praises the - antinomic - infinite sequence of ordinal numbers as "one of Cantor's boldest creations".

Later, axiomatic systems were developed that also allow infinite sequences, but ZFC is still established as the most important one.

7. Comment

Abraham Fraenkel, one of the founders of ZFC, demanded validity of set theory not only for abstract elements but also for "apples, pears and apricots". But the "empty set" of apples is contradictory. The set without apples cannot enlarge a set of apples, as demanded by the abstract theory (Article IV). Consistency with reality was failed to check.

Insisting on an allegedly consistent theory of the transfinite, which is accepted as contradicting reality, has a completely different dimension. **A disregard for scientific criteria restrains a discipline that Carl Friedrich Gauss crowned as the queen of the sciences.**

To refute this statement, reference could be made to the acceptance of imaginary numbers. They are not detectable in reality but are nevertheless suitable for its description, e.g. in electricity and quantum theory. That is correct, but only quantities that result from squaring the mathematical terms appear in measurements, so that imaginary numbers no longer appear. These are only helpful for efficient formulation. Transfinite numbers and sets, on the other hand, can never be brought into connection with reality and are nowhere helpful.

The sum of the arguments presented proves a grievous error in the thinking of the mathematical theorists of foundations. **The foundational crisis of mathematics was not overcome by measures intended to remedy it, but on the contrary was exacerbated.**

The author has submitted articles for publication in scientific journals demonstrating the contradictions of the foundations. They were rejected without comment or by wrong allegations. The discourse is denied. **Facts that refute false dogmas are ignored by the official representatives of mathematics and mathematical logic.**

These statements also apply to the 0, which is defined in a contradictory manner (Article I).

It is remarkable that the arguments about the meaning of zero coincide with the controversy about the helio-geocentric worldview. It was already known in antiquity that the earth revolves around the sun. Due to reservations in philosophy and religion, this knowledge was suppressed and the Ptolemaic system was maintained. Astronomical facts then led to the Copernican revolution, which was initially massively suppressed by the Catholic Church and culminated in the trial of the inquisition against Galilei.

Analogously, the meaning of 0, "nothing" and "no number", was established in antiquity before

reservations in philosophy and religion caused the contradictory reversal into "something" and "number". Empirical facts prove that the original meaning is true.

The previous behavior of mathematical officials described above resembles that of the priesthood of the seventeenth-century inquisition rather than that of modern scientists.

The exceedance of Gödel's incompleteness theorems is also affected by the verdict. The proof is based on a new axiom (Article IX) that states the equivalence of the existence of "nothing" of proof² to "non-existence" of a proof.

The refusal to retrace or examine evolutionary logic also reveals a crisis in thinking.

The author's appreciation for mathematics and mathematical logic, as well as - admittedly somewhat irritated - for mathematicians and logicians is not vitally affected thereby. The **unofficial statements** of two professors contributed to this. **"I wish you success with your 'nothing' attack" and "I am convinced that you are right. But I don't want to comply with your proposal for a cooperation. I don't want to expose myself to the slashing and stabbing that will follow."**

But the turning point in the foundational theory cannot be impeded on the basis of the facts presented. The elimination of contradictions and false metaphysical assumptions substantially simplifies the theory. Mathematics and logic are strengthened.

2 The 0 not only represents „nothing“ of numbers, i.e. „no number“, but also „nothing“ of proof, i.e. „no proof“.